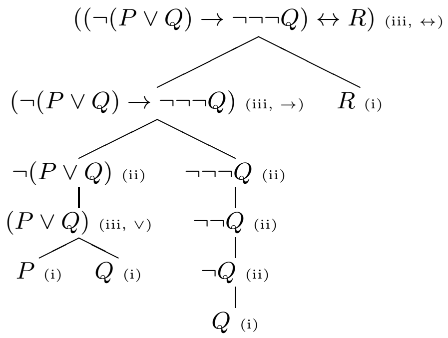
Syntax Let L_p be the language of propositional logic. The vocabulary of L_p comprises (i) a set of *proposition symbols* P, Q, R..., (ii) a unary connective $\neg$, (iii) binary connectives $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, and (iv) parenthesis (&).

The **well formed formulae** (wffs) of L_p are given by :

(i). All proposition symbols are wffs.
(ii). If φ is a wff of L_p, then $\neg\varphi$ is also a wff of L_p.
(iii). If φ and ψ are wffs of L_p, then so are $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$, and $(\varphi \leftrightarrow \psi)$.
(iv). Nothing else is a wff
(Nothing that cannot be constructed by successive steps of (i), (ii) or (iii) is a wff).



Semantics Let V be a *truth assignment* (or *valuation*) that maps all proposition symbols to a truth value (it can also be seen as a *model*). Then the truth value of any proposition can be defined/computed inductively as follows :

(i). If φ is a proposition symbol, then $[\![\varphi]\!]_V = V(\varphi)$;
(ii). If φ is a wff, then $[\![\neg\varphi]\!] = 1$ if and only if $[\![\varphi]\!] = 0$;
(iii). If φ and ψ are wffs, then
— $[\![(\varphi \wedge \psi)]\!] = 1$ iff $[\![\varphi]\!] = 1$ and $[\![\psi]\!] = 1$;
— $[\![(\varphi \vee \psi)]\!] = 0$ iff $[\![\varphi]\!] = 0$ and $[\![\psi]\!] = 0$;
— $[\![(\varphi \rightarrow \psi)]\!] = 0$ iff $[\![\varphi]\!] = 1$ and $[\![\psi]\!] = 0$;
— $[\![(\varphi \leftrightarrow \psi)]\!] = 1$ iff $[\![\varphi]\!] = [\![\psi]\!]$;

φ	$\neg\varphi$
0	1
1	0

φ	ψ	$\varphi \wedge \psi$
0	0	0
0	1	0
1	0	0
1	1	1

φ	ψ	$\varphi \vee \psi$
0	0	0
0	1	1
1	0	1
1	1	1

φ	ψ	$\varphi \rightarrow \psi$
0	0	1
0	1	1
1	0	0
1	1	1

φ	ψ	$\varphi \leftrightarrow \psi$
0	0	1
0	1	0
1	0	0
1	1	1

Properties of formulae A formula may be :

a tautology	always true
a contradiction	always false
contingent	

These properties can be checked by computing the full truth table for the formula.

Relations between formulae

• Two formulae φ and ψ may be :

contradictory	φ is true when ψ is false and vice-versa
contrary	φ and ψ are never true together (but may be false)
logically equivalent	φ and ψ always have the same truth value

• A formula ψ is a logical consequence of φ if : every time φ is true,, ψ is also true. (We also say that φ entails ψ).

These relations can be determined by computing the values of the two formulae in the same (full) truth table.