

TD8 - (Un)typed Lambda calculus

May 22, 2026

1. Are the following λ -terms well formed?

- (a) $(x) x.x$
- (b) $\lambda x.\lambda y.\lambda z.u$
- (c) $\lambda y.(\lambda x.(y)) x$
- (d) $\lambda x.(x x)$
- (e) $(x) \lambda y.x$

2. Reduce as much as possible the following λ -terms:

- (a) $(\lambda x.(x) x) \lambda x.x$
- (b) $((\lambda x.\lambda y.(y) x) f) \lambda x.x$
- (c) $((\lambda f.\lambda x.(f) (f) x) \lambda x.(\text{tall}) x) \text{student}$
- (d) $((\text{compose}) \lambda x.(\text{not}) x) \lambda y.(\text{tired}) y) \text{john}$, where $\text{compose} \triangleq \lambda f.\lambda g.\lambda x.(f) (g) x$
- (e) $((\text{and_pred}) \lambda x.(\text{smart}) x) \lambda y.(\text{kind}) y$, where $\text{and_pred} \triangleq \lambda P.\lambda Q.\lambda x.(P) x \wedge (Q) x$

3. Identify all redexes in the following term and reduce it as much as possible:

$$((\lambda S.\lambda V.(S) (V) \lambda Q.(Q) m) \lambda P.(P) j) \lambda O.\lambda y.(O) \lambda z.((\text{kiss}) y) z$$

4. For each sentence below, draw the syntactic tree and give the composition rules so that each fragment containing them is fully defined syntactically and compositionally.

- (a) Sam likes Pam.
- (b) Sam sings and runs.
- (c) Sam and Pam sleep.
- (d) A tabby cat meows.