TD6 - First order logic I

April 17, 2026

1. For each of the following predicate calculus formulas, indicate the scope of quantifiers and the free variables and specify whether it is a sentence.

   (a) $\exists x\ (Axy \wedge Bx)$
   (b) $\exists x\ Axy \wedge Bx$
   (c) $\exists x \exists y\ Axy \rightarrow Bx$
   (d) $\exists x\ (\exists y\ Axy \rightarrow Bx)$
   (e) $\neg \exists x \exists y\ Axy \rightarrow Bx$
   (f) $\forall x \neg \exists y\ Axy$
   (g) $\neg Bx \rightarrow (\neg \forall y\ (\neg Axy \vee Bx) \rightarrow Cy)$

   (a) $\exists x\ (Axy \vee By)$
   (b) $\exists x\ Axx \vee \exists y\ By$
   (c) $\exists x\ (\exists y\ Axy \vee By)$
   (d) $\forall x \forall y\ ((Axy \wedge By) \rightarrow \exists w\ Cxw)$
   (e) $\forall x\ (\forall y\ Ayx \rightarrow By)$
   (f) $\forall x \forall y\ Ayy \rightarrow Bx$

2. [*Homework 23/24*] Translate as precisely as possible the following sentences into predicate logic. Explain the interpretation of non logical constants when it is not obvious. In case of ambiguity, propose as many formulae as necessary.

   (a) Every politician is rejected by some voters.

   (b) Alex is upset as soon as everyone is noisy.

   (c) A child is confident only if no adult lies to him.

   (d) The moon has exactly two satellites.

   (e) When a problem is solved by all students, it should be suppressed.

   (f) All the newspapers which don't have readers will disappear if they don't find a buyer.

   (g) Either everyone takes a drink, or no one does.

   (h) Someone who refuses that everybody loves her should consult a doctor.

3. [*Exam 23/24*] Translate as precisely as possible the following sentences into predicate logic. In case of ambiguity, provide a formula for each possible reading.

   (a) An accomplished person doesn't talk unless everyone listens.

   (b) Not all the guests appreciated a singer.

   (c) Everytime Mia finds a wallet she gives it back to its owner.

   (d) At least one person has to come to make Sam happy.

4. [*Homework 22/23*] Among the formulae given in (1), indicate those which are appropriate representations for (2). Provide an explanation for those which are not appropriate.

   (1) a. $\forall x\ ((Px \wedge \exists y\ (Cy \wedge Rxy)) \rightarrow Sxy)$
   b. $\forall x \forall y\ ((Px \wedge Cy \wedge Rxy) \rightarrow Sxy)$
   c. $\forall x\ (Px \rightarrow \forall y\ ((Cy \wedge Rxy) \rightarrow Sxy))$
   d. $\forall x \exists y\ ((Px \wedge Cy \wedge Rxy) \rightarrow Sxy)$

   (2) Any person who raises a child in sacrificing for them.

   Note: Sxy means x is sacrificing for y.

5. [*Homework 23/24*] Construct an analysis of *all but one* in terms of $\forall$, $\exists$, $=$, and propositional connectives, and show how (3) would be analysed under your proposal.

   (3) All but one poet hated himself.

6. [*Homework 22/23*] Let us consider the following syllogism:

If a student fails, they didn't work correctly
Alex is a student
Alex failed
∴ Alex didn't work correctly

(a) Translate this syllogism into predicate logic.

(b) Give the simplest possible model in which all the propositions are true.

(c) A model-theoretic proof for this syllogism would try to prove that every model that makes the premises true would also satisfy the conclusion. Sketch informally a proof of this sort.