

TD5 - Propositional logic

April 10, 2026

1. Among the following expressions, which are well-formed formulae of L_p ?

(1) $\neg(\neg P \vee Q)$	(5) $(P \rightarrow ((P \rightarrow Q)))$	(9) $(P \vee (Q \vee R))$
(2) $P \vee (Q)$	(6) $((P \rightarrow P) \rightarrow (Q \rightarrow Q))$	(10) $\neg P \vee Q \vee R$
(3) $\neg(Q)$	(7) $((P_{28} \rightarrow P_3) \rightarrow P_4)$	(11) $(\neg P \vee \neg\neg P)$
(4) $(P_2 \rightarrow (P_2 \rightarrow (P_2 \rightarrow P_2)))$	(8) $(P \rightarrow (P \rightarrow Q) \rightarrow Q)$	(12) $(P \vee P)$
2. Show that the connectives \wedge and \neg are sufficient, *i.e.* any formula containing other connectives ($\vee, \rightarrow, \leftrightarrow$) is equivalent to a formula having only \wedge and \neg .
3. Alice, Bob and Charlie are suspected of tax fraud. They provide the following testimonies:

Alice: Bob is guilty and Charlie is innocent.
 Bob: If Alice is guilty, Charlie is guilty too.
 Charlie: I am innocent but at least one of the other two is guilty.

Express each suspect's testimony in propositional logic. Answer the following questions, using a truth table:

- (a) Is it possible for all three testimonies to be true simultaneously?
- (b) If all suspects are innocent, is someone lying, and if so, who?
- (c) If the guilty ones lie and the innocent ones tell the truth, who is innocent and who is guilty?