

## TD3 - Formal grammars

March 27, 2026

1. What are the languages generated by the following grammars? Are the grammars ambiguous?
  - (a)  $\mathcal{G}_1 = (\{a\}, \{S\}, S, \{S \rightarrow Sa \mid a\})$
  - (b)  $\mathcal{G}_2 = (\{a, b\}, \{S, T\}, S, \{S \rightarrow Tb, T \rightarrow Ta \mid \varepsilon\})$
  - (c)  $\mathcal{G}_3 = (\{(\cdot)\}, \{S\}, S, \{S \rightarrow SS \mid (S) \mid \varepsilon\})$
2. Propose grammars that generate the following languages:
  - (a)  $L_1 = \{a^i b^j c^k \mid (i = j \text{ or } j = k) \text{ and } i, j, k \geq 1\}$
  - (b)  $L_2 = \{ww^R \mid w \in \{a, b\}^*\}$
  - (c)  $L_3 = \{w \in \{a, b\}^* \mid |w|_a \neq |w|_b\}$
  - (d) The set of valid arithmetic expressions (in  $\mathbb{N}$ ) over  $\{0, 1, \dots, 9, \times, +, (, )\}$
  - (e) [Exam 22/23]  $L_4 = \{ab^n cb^n a \mid n \geq 0\}$
3. [Homework 23/24]
  - (a) Let  $G$  be the grammar  $S \rightarrow aSbb \mid \varepsilon$ . Describe informally the language it generates.
  - (b) Let  $G'$  be the grammar  $S' \rightarrow SSS, S \rightarrow aSbb \mid \varepsilon$ , with  $S'$  the start symbol (axiom). Describe informally the language it generates.
  - (c) Let  $G_1$  and  $G_2$  be context-free grammars, and  $L(G_1)$  and  $L(G_2)$  the languages they generate. How do you form context-free grammars generating the following sets:
    - i.  $L(G_1) \cup L(G_2)$
    - ii.  $L(G_1)L(G_2)$
    - iii.  $L(G_1)^*$