Ex 1:

(a) $\mathcal{L}(\mathcal{G}_1) = \{ a^n \mid n \geq 1 \}$

Not ambiguous: there is only one way of creating words and it is by pre-pending 'a'.

(b) $\mathcal{L}(\mathcal{G}_2) = \{ a^n b \mid n \geq 0 \}$

Not ambiguous: analogous do (a)

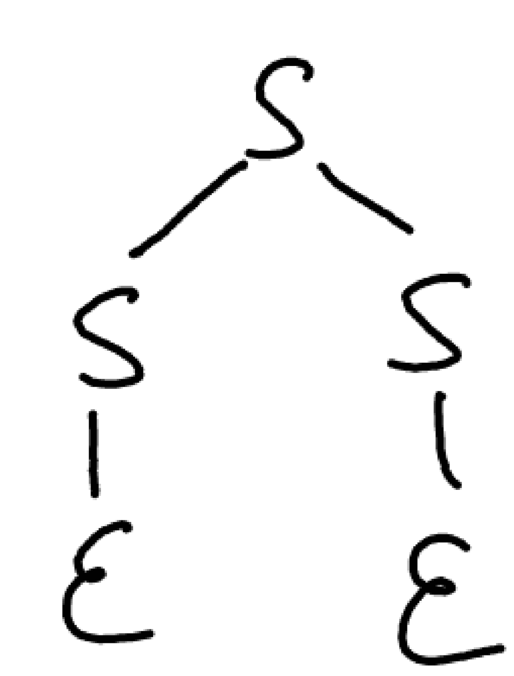
(c) $\mathcal{L}(\mathcal{G}_3) = \{ w \in \Sigma^* \mid \forall u \in \Sigma^*, (\exists v \in \Sigma^*, w = uv) \longrightarrow |u|_{(} \geq |u|_{)} \}$

This is Dyck (-1) language: the set of words such that any prefix contains at least as many '(' as ')'

$\longrightarrow$ In short: the set of words with balanced parentheses

Ambiguous: take $w = \varepsilon$

and



Unambiguous variant $\mathcal{G}_3'$: $S \longrightarrow (S)S \mid \varepsilon$

Ex 2:

(a) $S \to A S_1 \mid S_2 C$

$S_1 \to b S_1 c \mid bc$

$S_2 \to a S_2 b \mid ab$

$A \to aA \mid a$

$C \to cC \mid c$

(b) $S \to aSa \mid bSb \mid \varepsilon$

(c) $S \to A \mid B$

$A \to aAbA \mid bAaA \mid aA \mid a$

$B \to bBaB \mid aBbB \mid bB \mid b$

$S \to A \mid B$

$A \to TaA \mid TaT$

$B \to TbB \mid TbT$

$T \to aTbT \mid bTaT \mid \varepsilon$

(d) $E \to (E) \mid E \times E \mid E + E \mid N$

$N \to 0 \mid 1N' \mid \cdots \mid 9N'$

$N' \to 0N' \mid 1N' \mid \cdots \mid 9N'$

OR

$N \to 1N' \mid \cdots \mid 9N'$
$\mid 0 \mid 1 \mid \cdots \mid 9$

(e) $S \to aXa$

$X \to bXb \mid c$

Ex 3:

(a) $L(\mathcal{G}) = \{a^k b^{2k} \mid k \geq 0\}$

(b) $L(\mathcal{G}') = L(\mathcal{G})^3$

(c) $\mathcal{G}_1 = (\Sigma, V_1, S_1, R_1)$

$\mathcal{G}_2 = (\Sigma, V_2, S_2, R_2)$

wlog $V_1 \cap V_2 = \emptyset$ (otherwise, rename $V_2 \to V_2'$)

$\mathcal{G}_3 = (\Sigma, S \cup V_1 \cup V_2, S, R_1 \cup R_2 \cup R)$

with $S \notin V_1 \cup V_2$ (otherwise, rename vars)

(i) $R = \{S \to S_1 \mid S_2\}$

(ii) $R = \{S \to S_1 S_2\}$

(iii) $R = \{S \to S_1 S \mid \varepsilon\}$