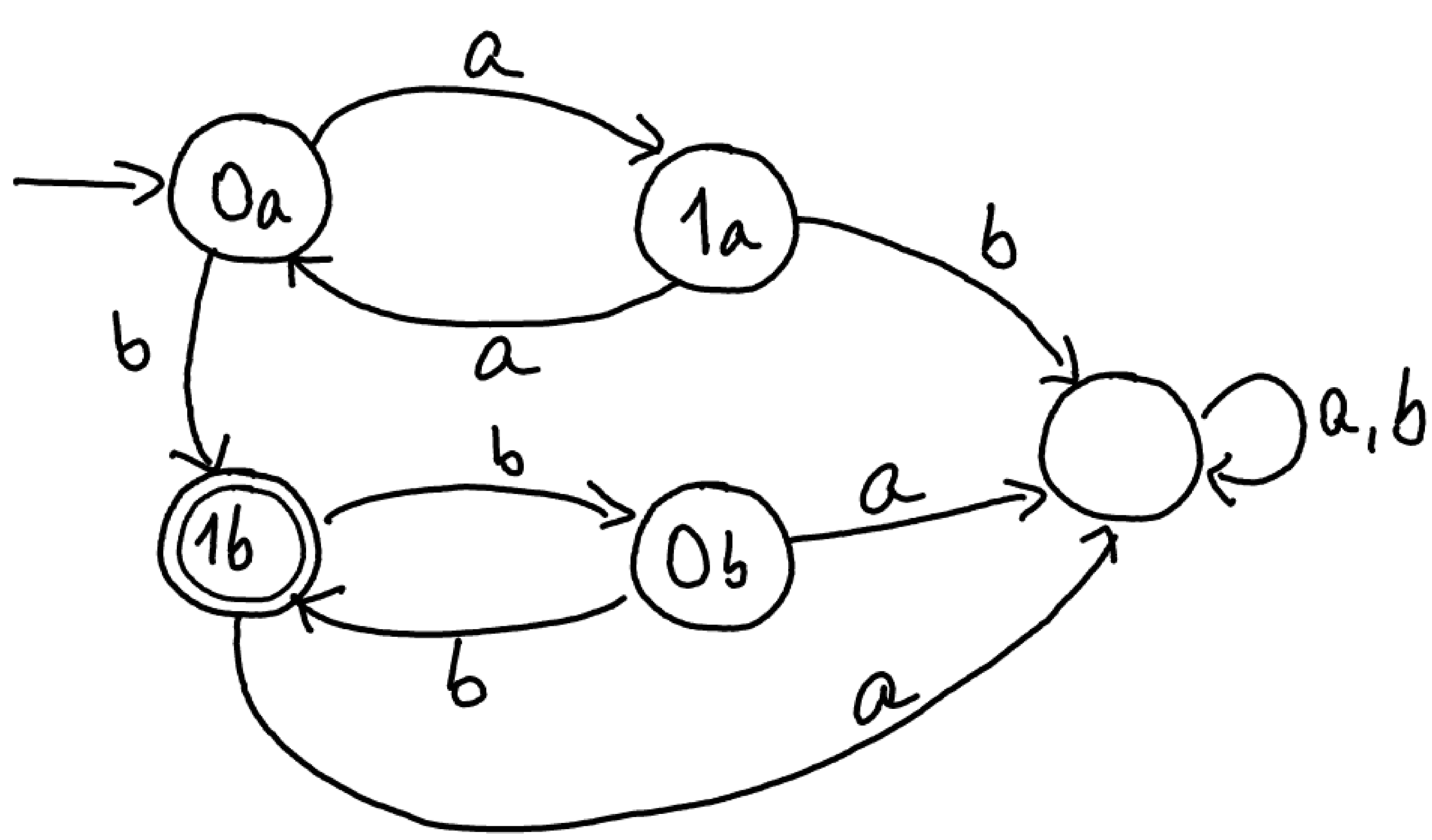
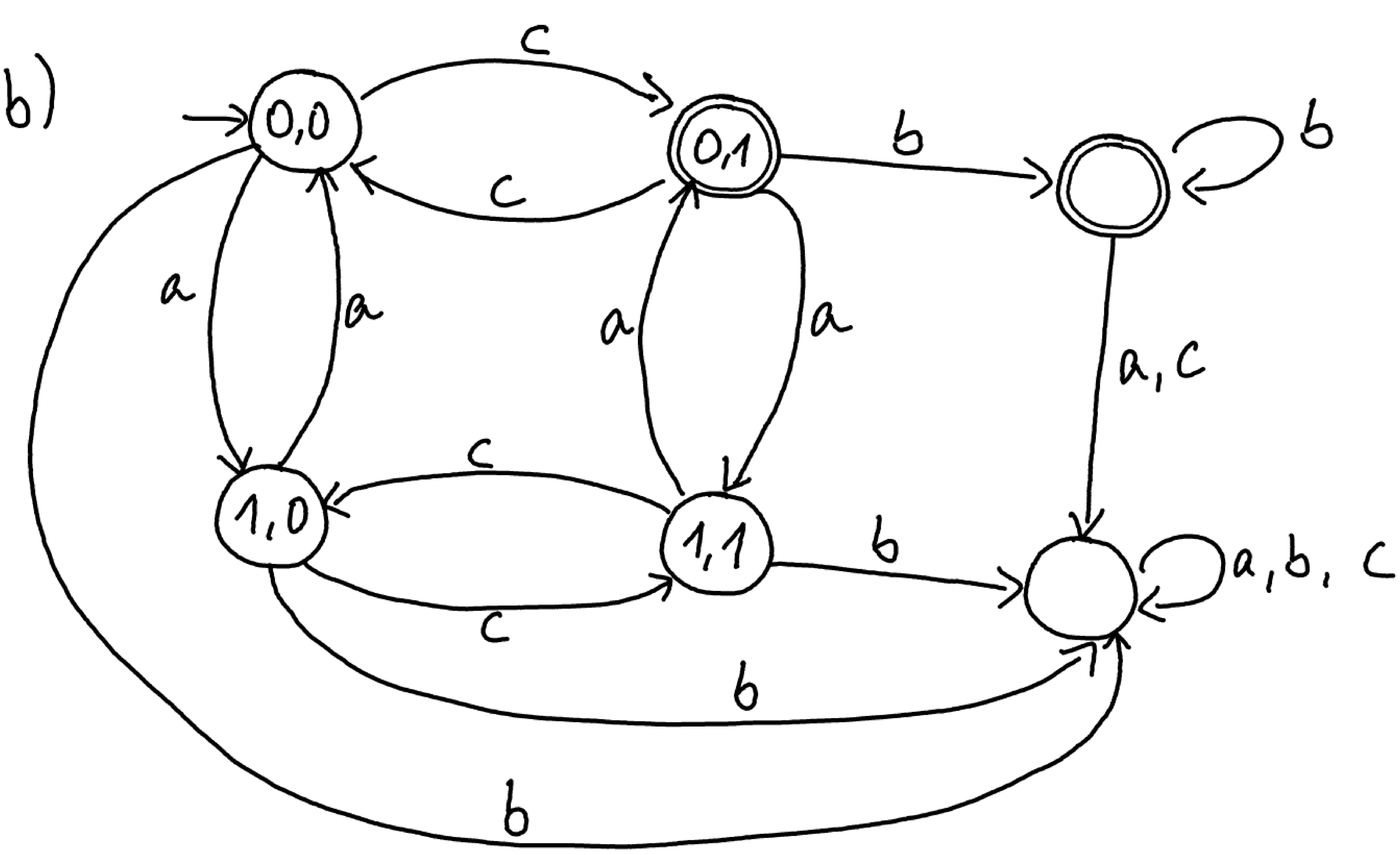
Ex 1:

(a)

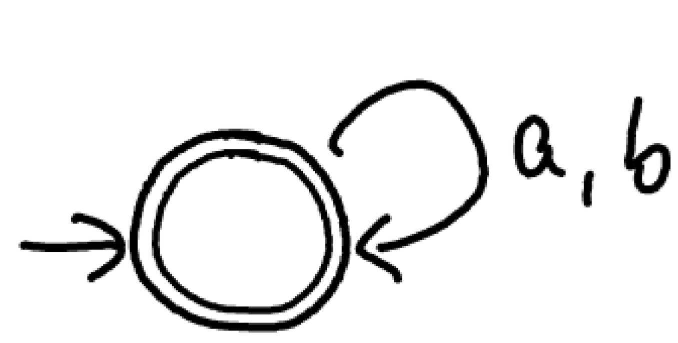


(b)



Ex 2:

(a)

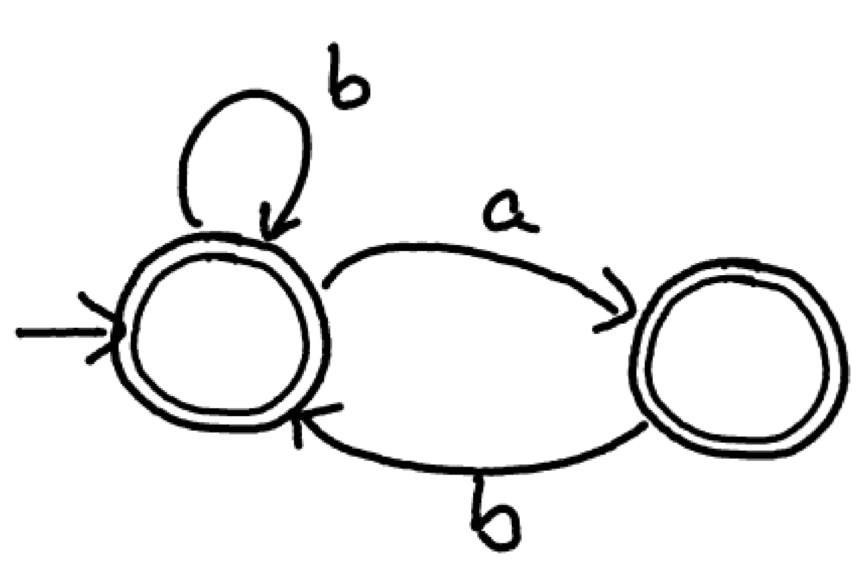


L: any word over {a, b}

(b)

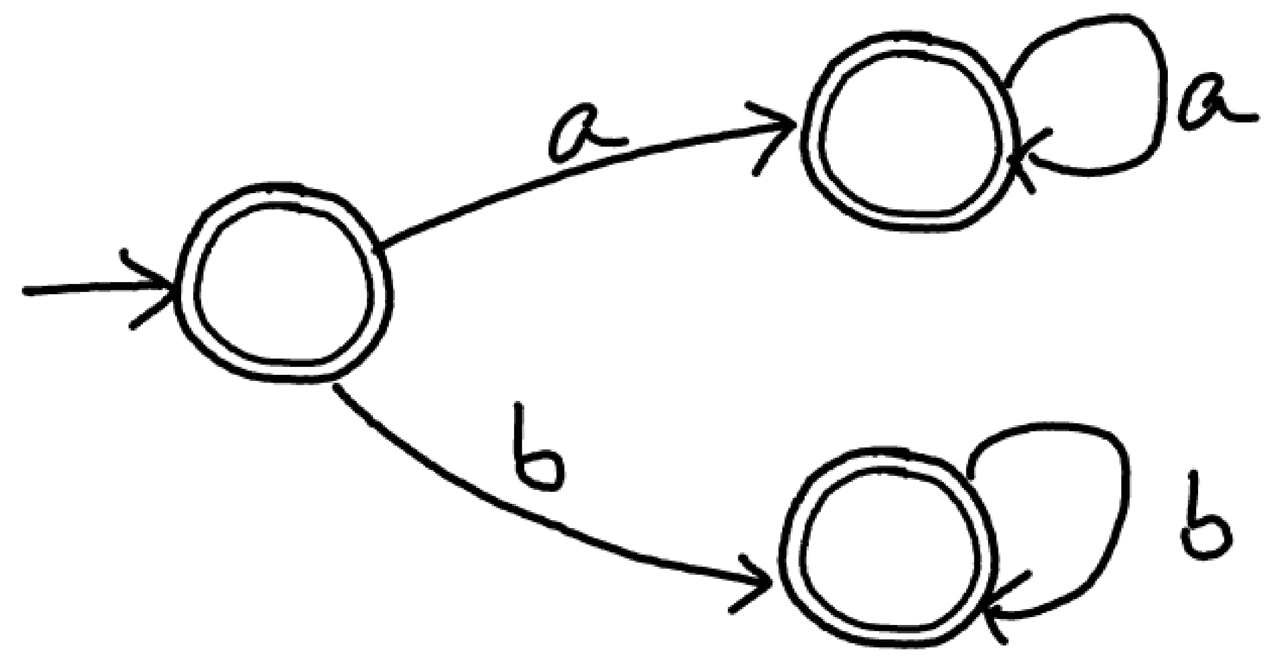
L: words beginning with 'a'

(c)



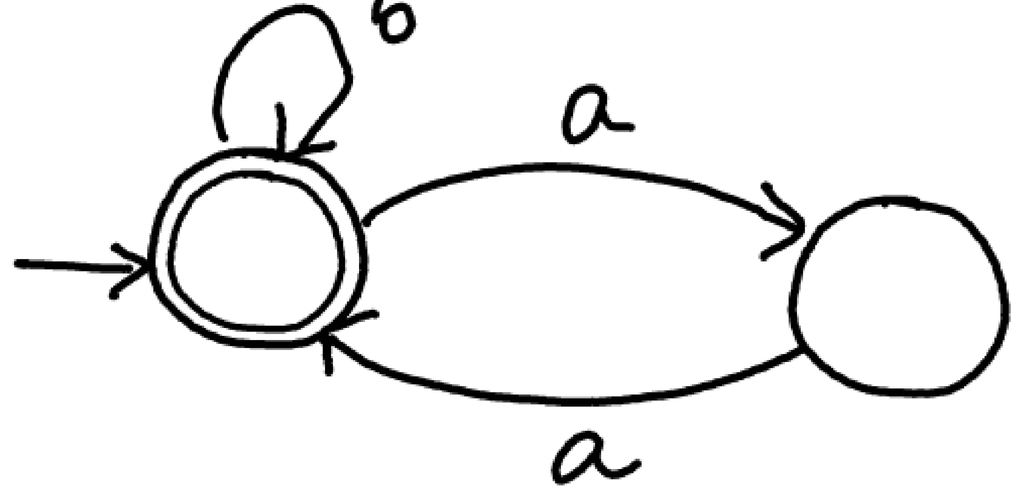
L: words without any two consecutive 'a'

(d)



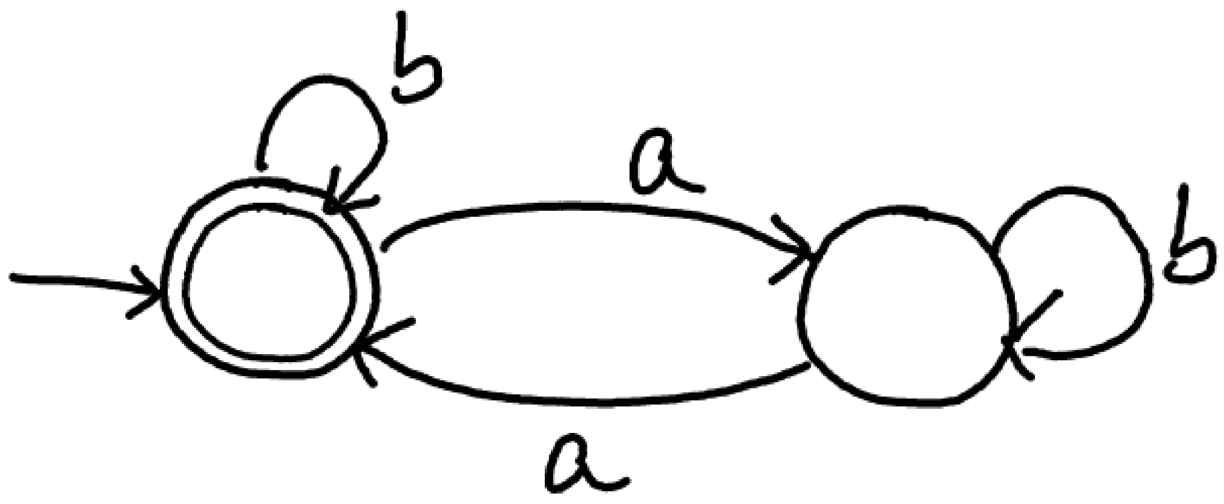
L: words with only 'a' or 'b'

(e)



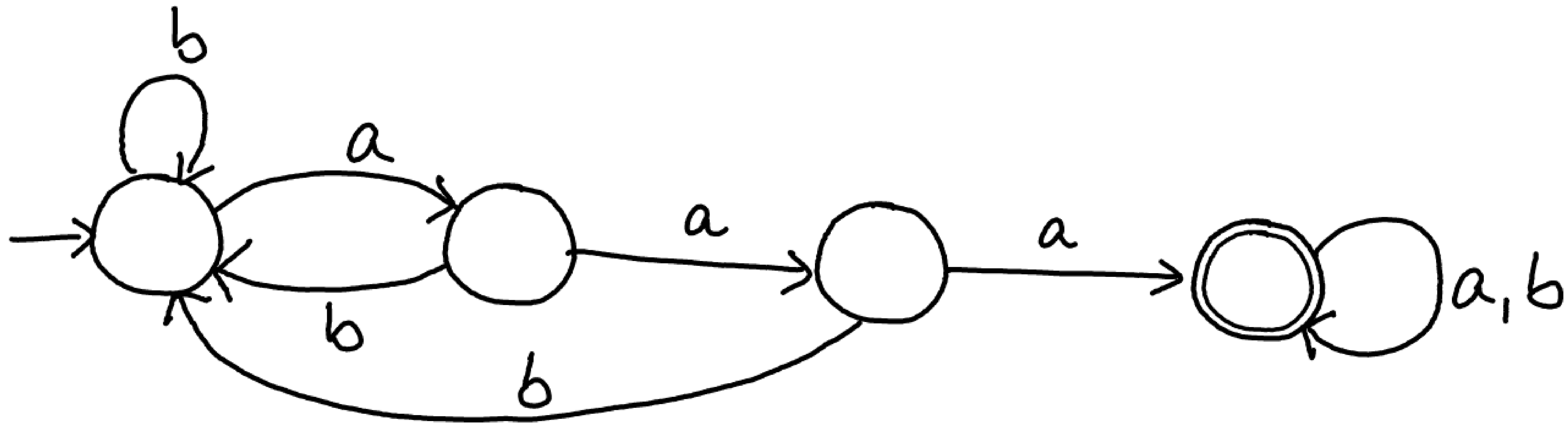
L: words where any 'a' appears in pairs

(f)



L: words with an even number of 'a'

(g)



L: words with a group of three consecutive 'a'

Ex 3:

(a) $(a|b|c)(a|b|c)^*$

(b) $\varepsilon + (a+b+c)^*(a+c)$

(c) $(a|c)^* b (a|b|c)^*$

(d) $(a+c)^* (\varepsilon | b(a+c)^*)$

(e) $((a+b+c)(a+b+c))^* ab$ or $((a+b+c)^2)^* ab$

(f) $(a|b+c)^*(a|c)(a|b|c)(a|b|c)$ or $(a|b|c)^*(a|c)(a|b|c)^2$

Ex 4:

(a) $L_1 = \{ ab^n cd^n e \mid n \in \mathbb{N} \}$ rational?

The exponents of 'u' and 'v' being the same, there is a memory requirement in the model recognising this language the DFA's don't have, so the intuition is that L_1 is not rational. Let us prove this with the pumping lemma.

By contradiction, suppose L_1 is regular. Then, by the pumping lemma, it verifies the pumping property. Let $k > 0$ be the pumping length. Take $w = ab^k cd^k e$.

Consider its decomposition $w = xuy$:

(i) $|u| \geq 1$

(ii) $|xu| \leq k$ so y is of the form $b^m cd^k e$, with $1 \leq m \leq k$, and xu is of the form ab^{k-m}.

There are two cases:

- $\underline{m=k}$: Then, $xu = ab^0 = a$. This way, $x = \varepsilon$ and $u = a$, therefore

  (iii) $xu^0y = xy = \varepsilon\, bcd^1e = bcde$ should be in L_1.

- $\underline{m<k}$: Then, xu includes at least a 'b', especially u, therefore

(iii) $xu^0y = xy$ would be of the form ab^rcd^ke or cd^ke, with $r<k$, and not be part of L_1.

In both cases, we have a contradiction. Hence, L_1 is not regular

(b) $L_2 = \{a^m b^n \mid n \equiv m \pmod 3\}$ rational?

n and m have the same remainder when divided by 3. There are 3 cases according to the remainder value: 0, 1 and 2:

$$L_2 = (a^3)^*(b^3)^* + a(a^3)^* b(b^3)^* + a^2(a^3)^* b^2(b^3)^*$$

(c) $L_3 = \{w \in \{a,b\}^* \mid |w|_a < |w|_b\}$ rational?

By contradiction, suppose L_3 is regular. Let $p>0$ be the pumping length. Consider $w = a^p b^{p+1}$. For any decomposition $w = xuy$ with $|u| \geq 1$ and $|xu| \leq p$ we have $u = a^d$ with $0<d\leq p$. Hence, $xu^2y = xuuy = a^{p+d} b^{p+1}$ s.t. $|w|_a \geq |w|_b$ (since $d \geq 1$) should be in L_3. Contradiction

(d) $L_4 = \{ w \in \{a,b,c\}^* \mid (|w|_a = 0) \Rightarrow (|w|_b = 0) \}$ rational?

Let us leverage the antecent in the implication. If $w \in L_4$ does not have any 'a', then it does not have any 'b' either. <u>However</u>, if w does have at least an 'a', then there is no condition on the number of 'b'.

$$L_4 = c^* + (b+c)^* a (a+b+c)^*$$

(e) $L_5 = \{ ww \mid w \in \{a,b\}^* \}$ rational?

By contradiction, suppose L_5 is regular. Let $p > 0$ be the pumping length. Consider $w = a^p b a^p b$. If we decompose $w = xuy$ such that $|u| \geq 1$ and $|xu| \leq p$, then

- $u = a^d$ with $1 \leq d \leq p$ $\quad(|u| = d \geq 1)$
- $x = a^r$ with $0 \leq r \leq p-d$ $\quad(|xu| = d + r \leq p)$
- $y = a^{p-d-r} b a^p b$
- $xu^0y = xy = a^r a^{p-d-r} b a^p b = a^{p-d} b a^p b$,

with $p-d < p$, is not part of L_5. <u>Contradiction</u>

(f) $L_6 = \{ w_1 w_2 \mid w_1, w_2 \in \{a,b\}^* \text{ and } |w_1| = |w_2| \}$ rational?

The words in this language can be split into two subwords of equal length. In other terms, any word of this language has even length!

As we saw in the previous tutorial, L_6 is rational.

$$L_6 = ((a|b)^2)^* = (aa|ab|ba|bb)^*$$