TD1 - Introduction to Formal Languages

March 13, 2026

1. Infer and describe in plain English the corresponding languages:

   (a) $L_1 = \{c, ac, abc, bc, bac, baabaac, ababac, \ldots\}$ over $\Sigma_1 = \{a, b, c\}$.

   Solution: L_1 is the set of words ending with a c. Formally, $L_1 = \{wc \mid w \in \Sigma_1^*\}$. Arguably, a more restrictive language could be inferred from the sequence of words, that is, the language with a **single** and final c: $L_1 = \{wc \mid w \in \{a, b\}^*\}$.

   (b) $L_2 = \{\varepsilon, a, ab, aabb, aaa, aaabb, aaaab, \ldots\}$ over $\Sigma_2 = \{a, b\}$.

   Solution: L_2 is the set of words composed of a's followed by a lesser number of b's. Formally, $L_2 = \{a^m b^n \mid m \geq n \geq 0\}$.

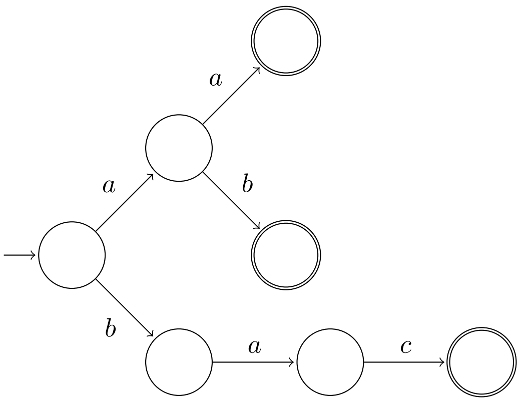
   (c) $L_3 = \{ab, abab, aaab, baab, bbbbab, \ldots\}$ over $\Sigma_3 = \{a, b\}$.

   Solution: L_3 is the set of words with even length and ending with ab. Formally, $L_3 = \{wab \mid |w| \equiv 0 \pmod 2\}$

2. Let $\Sigma = \{a, b, c\}$. Give finite-state automata that recognise the following languages:

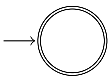
   (a) $L_1 = \{aa, ab, bac\}$.

   Solution: Here, I show one of the simplest **deterministic** finite-state automata (FSA): having a final state for each word in the language. As we saw during the tutorial, simpler FSAs are possible, notably with fewer states.



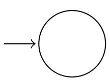
   (b) $L_2 = \{\varepsilon\}$.

   Solution:



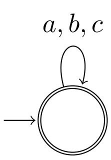
   (c) $L_3 = \emptyset$.

   Solution:



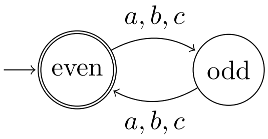
   (d) $L_4 = \Sigma^*$.

   Solution:



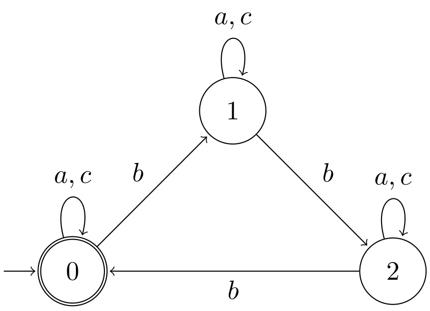
3. Let $\Sigma = \{a, b, c\}$. Give finite state automata that accept the following languages:
   (a) The set of words with even length.

   Solution:

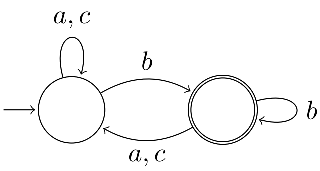
   (b) The set of words where the number of occurrences of b is divisible by 3.

   Solution:

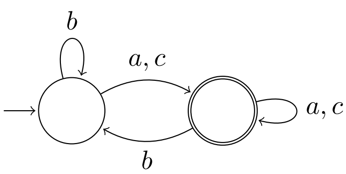
   (c) The set of words ending with a b.

   Solution:

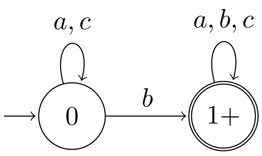
   (d) The set of non-empty words not ending with a b.

   Solution:

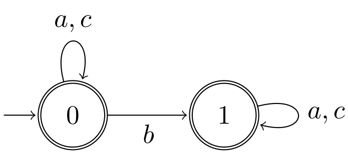
   (e) The set of words containing at least a b.

   Solution:

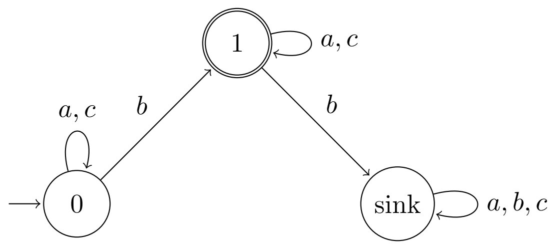
   (f) The set of words containing at most a b.

   Solution:

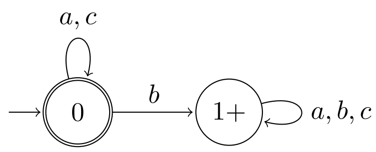
(g) The set of words containing exactly one b.

Solution: I added a 'sink' state, although it was not necessary to include it.



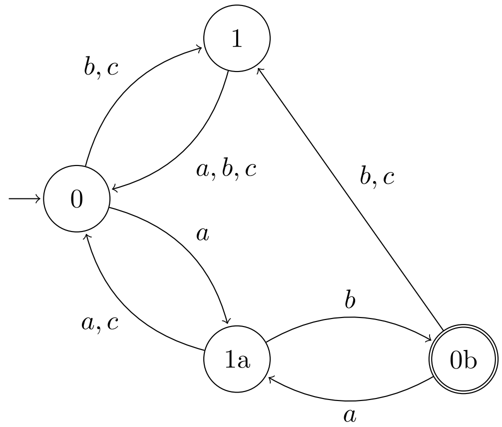
(h) The set of words not containing any b.

Solution:



(i) The set of words with even length ending with ab.

Solution: Here, I use 0 for 'even' states, and 1 for 'odd' ones.



4. Let $\Sigma = \{a, b\}$. Let $L_1 = \{a, ab, ba\}$ and $L_2 = \{\varepsilon, b, ab\}$ be two languages.

(a) Compute the following operations:

$$L_1.L_2 \quad L_2.L_1 \quad L_2 \setminus L_1 \quad L_1.\{\varepsilon\} \quad \emptyset.L_2$$

Solution:

- $L_1.L_2 = \{a, ab, aab, abb, abab, ba, bab, baab\}$
- $L_2.L_1 = \{a, ab, ba, bab, bba, aba, abab, abba\}$
- $L_2 \setminus L_1 = \{\varepsilon, b\}$
- $L_1.\{\varepsilon\} = L_1 = \{a, ab, ba\}$
- $\emptyset.L_2 = \emptyset$

(b) If $L_3.L_4 = \{\varepsilon\}$, what can be said about languages L_3 and L_4?

Solution: $L_3 = L_4 = \{\varepsilon\}$

(c) If $L_3.L_4 = \emptyset$, what can be said about languages L_3 and L_4?

Solution: $L_3 = \emptyset$ or $L_4 = \emptyset$