HW2 - First-order logic

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1. For each of the following predicate logic formulae, number the (different) occurrences of the variables with subscripts according to their scope. For instance,

$$\exists x\ (\exists y\ Axyz \to Bxyz)$$

would become

$$\exists x_1\ (\exists y_1\ Ax_1y_1z_1 \to Bx_1y_2z_1).$$

   (a) $\exists x\ (Axy \vee By)$

   (b) $\exists x\ (Axx \wedge \forall x\ Bxy) \vee \exists y\ Cy$

   (c) $\exists x\ (\exists y\ Axy \vee By)$

   (d) $\forall x \forall y\ ((Axy \wedge By) \to \exists w\ Cxw)$

   (e) $\forall x\ (\forall y\ Azx \to By)$

   (f) $\forall x \forall y\ Ayy \to Bx$

2. Simplify the following expression:

$$(q \to (p \wedge r)) \to (p \to q)$$

3. Translate as precisely as possible the following sentences into predicate logic. If ambiguous, provide a formula for each possible reading.

   (1) a. All superheroes had a difficult time.
   b. Every time Mia finds a wallet she gives it back to its owner.
   c. All newspapers which don't have readers will disappear if they don't find a buyer.
   d. Every student who solves a problem will explain it.
   e. There are only two solutions to every problem.
   f. Everyone is marked by an unrequited love.
   g. Nobody who is not a French likes cheese better than any French.
   h. Only completely consistent people are dead.

4. Consider a language that includes binary predicates P and equality (=). Consider the three following formulae:

$$(F_1)\ \forall x \forall y\ (Pxy \to x \neq y) \qquad (F_2)\ \forall x \forall y\ (Pxy \wedge x \neq y) \qquad (F_3)\ \forall x \exists y\ (Pxy \wedge x \neq y)$$

For each formula, determine whether there exists a model $\mathcal{M} = \langle D, I \rangle$ that satisfies it:

   (a) when D is a singleton,

   (b) when D contains exactly two elements,

   (c) when $D = \mathbb{N} = \{0, 1, 2, \dots\}$ and $I(P) = \{(x, y) \mid x \text{ is divisible by } y\}$.

5. (a) Translate the following sentences into predicate logic.

   (2) a. John owns everything he has not lost.
   b. John has not lost 1 million dollars.
   c. John owns one million dollars.

   (b) Analyse the syllogism going from the conjunction of (2a) and (2b) to the conclusion (2c). Explain where the problem lies.