- Bound variables are "dummy" : their name do not matter.

$$\forall x\, Fx \quad \equiv \quad \forall y\, Fy$$

But beware of unintended captures :

$$\forall x\,(Fx \wedge Gy) \quad \not\equiv \quad \forall y\,(Fy \wedge Gy)$$

- Duality rules (*de Morgan* laws)

$$\begin{array}{rcl}
\forall x\ \alpha & \equiv & \neg\exists x\ \neg\alpha \\
 & \textit{for instance :} & \\
\forall x\ Rx & \equiv & \neg\exists x\ \neg Rx \\
\textit{All is relative} & \approx & \textit{Nothing is absolute } (\approx \textit{ non relative}) \\
\forall x\ (Px \to Kx) & \equiv & \neg\exists x\ (Px \wedge \neg Kx) \\
\textit{All professors are kind} & \approx & \textit{There are no non-kind professors} \\
 & \textit{Other variants :} & \\
\exists x\ \alpha & \equiv & \neg\forall x\ \neg\alpha \\
\neg\exists x\ \alpha & \equiv & \forall x\ \neg\alpha \\
\neg\forall x\ \alpha & \equiv & \exists x\ \neg\alpha
\end{array}$$

- Distribution rules :

$$\boxed{\begin{array}{rcl}
\forall x\ (\alpha \wedge \beta) & \equiv & (\forall x\ \alpha \wedge \forall x\ \beta) \\
\textit{All is rare and expensive} & \approx & \textit{All is rare and all is expensive} \\
 & \textbf{\textit{But :}} & \\
\forall x\ (\alpha \vee \beta) & \not\equiv & (\forall x\ \alpha \vee \forall x\ \beta) \\
\textit{All is either relative or absolute} & \not\approx & \textit{Either all is relative or all is absolute}
\end{array}}$$

$$\boxed{\begin{array}{rcl}
\exists x\ (\alpha \vee \beta) & \equiv & (\exists x\ \alpha \vee \exists x\ \beta) \\
 & \textbf{\textit{But :}} & \\
\exists x\ (\alpha \wedge \beta) & \not\equiv & (\exists x\ \alpha \wedge \exists x\ \beta)
\end{array}}$$

$$\boxed{\exists x\ (\alpha \to \beta) \quad \equiv \quad (\forall x\ \alpha \to \exists x\ \beta)}$$

- Conditional distribution ($\bar{\beta}$ doesn't contain free occurrences of x)

$$\begin{array}{rcl}
\bar{\beta} & \equiv & \forall x \bar{\beta} \\
\bar{\beta} & \equiv & \exists x \bar{\beta} \\
 & & \\
\forall x\ (\alpha \vee \bar{\beta}) & \equiv & (\forall x\ \alpha \vee \bar{\beta}) \\
\exists x\ (\alpha \wedge \bar{\beta}) & \equiv & (\exists x\ \alpha \wedge \bar{\beta}) \\
\forall x\ (\alpha \to \bar{\beta}) & \equiv & (\exists x\ \alpha \to \bar{\beta}) \\
\textit{Every entity is such that if it breaks, there is noise} & \approx & \textit{If some entity breaks, there is noise} \\
\forall x\ (\bar{\beta} \to \alpha) & \equiv & (\bar{\beta} \to \forall x\ \alpha) \\
\textit{For all person, if there is noise, s/he is upset} & \approx & \textit{If there is noise, everyone is upset}
\end{array}$$