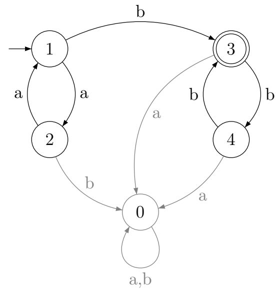
Automata

Ex. 1

Propose a complete deterministic finite state automaton which recognizes all the words that are composed of an even number of a's, followed by an odd number of b's ($\Sigma = \{a, b\}$).

Answer

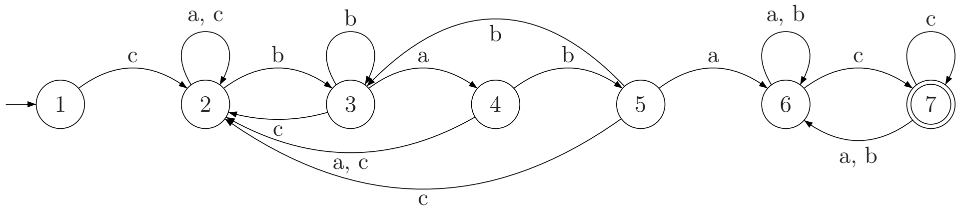


Ex. 2

Draw a deterministic automaton which recognizes all the words on the monoid $\{a, b, c\}^*$ which start with c, include the factor $baba$, and end with c.

Answer

Note: this automaton is deterministic but not complete. To get a complete version, a "well" state has to be inserted, with 2 transitions (a and b) from the initial state to it.



When state 6 is reached, the factor $baba$ has been recognized once. As a consequence, no additional constraint should be put on the rest of the word other than ending with c. In particular, going back to state 3 with a b (e.g.) would require the factor $baba$ to be recognized once again.

Formal Grammars

Ex. 3

Consider the following context-free grammar:

S	$\rightarrow$	p
p	$\rightarrow$	gn v1 *que* p | gn v2
gn	$\rightarrow$	np | det nc
np	$\rightarrow$	*Léa* | *Luc* | *Ève* | *Max*
nc	$\rightarrow$	*femme* | *homme* | *étudiante* | *étudiant* | *fille* | *garçon*
det	$\rightarrow$	*le* | *la* | *l'*
v1	$\rightarrow$	*pense* | *croit* | *voit* | *sait* | *dit* | *raconte*
v2	$\rightarrow$	*se promène* | *marche* | *part*

1. Give 4 distinct sentences that belong to the engendered language, containing respectively 0, 1, 2 and 3 times the word *que*.
2. For which reasons are these sentences not all well-formed in French? How can the grammar be amended to fix this?
3. Draw the derivation tree for *Luc sait que la femme croit que Léa part*.

. . . . Answer . . . .

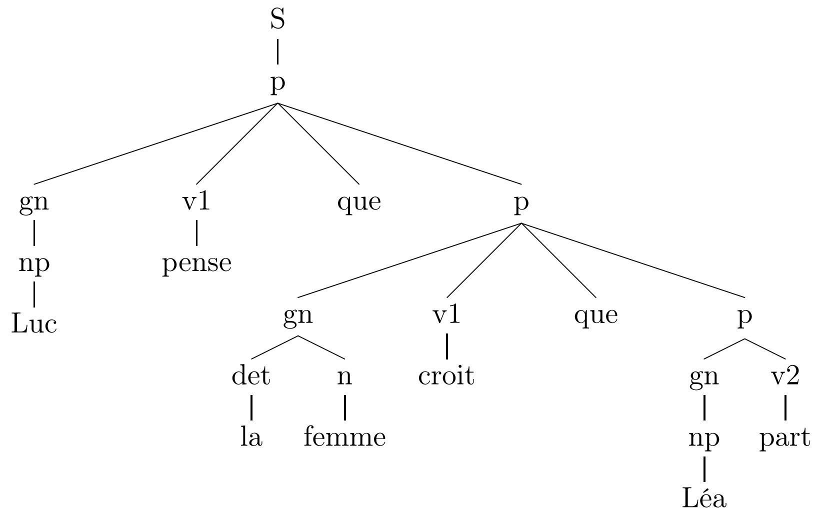
1. Léa se promène
   Luc voit que le homme part
   La femme pense que Léa croit que Ève marche
   Max sait que le fille raconte que Ève dit que Max marche

2. (a) Ellision: *le homme* instead of *l'homme*, and *que Ève* instead of *qu'Ève* (the two cases are similar, it was not asked that they were both solved).
   (b) Agreement: *le fille* instead of *la fille*.

   The easiest and most elegant way to solve those issues would be to equip the grammar with features that could be checked along with derivations. But it is also possible to simply use a modified version of the context-free grammar (only new rules are given below):

		Agreement			*Ellision (le/la ⇝ l')*
gn	→	np | det-m nc-m | det-f nc-f	gn	→	np | det-m nc-m | det-f nc-f | det-e nc-e
nc-f	→	*femme* | *étudiante* | *fille*	nc-e	→	*étudiante* | *homme* | *étudiant*
nc-m	→	*homme* | *étudiant* | *garçon*	nc-f	→	*femme* | *fille*
det-m	→	*le* | *l'*	nc-m	→	*garçon*
det-f	→	*la* | *l'*	det-e	→	*l'*
			det-m	→	*le*
			det-f	→	*la*

   (A similar treatment can be proposed for the case *que* ⇝ *qu'*.)

3. 

Ex. 4

Let's consider the sentence (1), which is well-known for its being syntactically ambiguous.

(1) Sam saw a girl with his telescope.

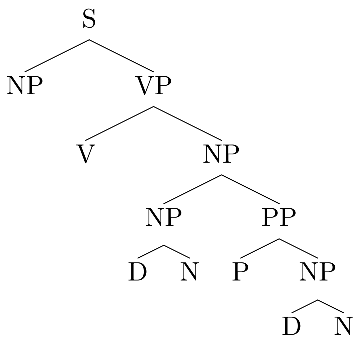
1. Show the syntactic ambiguity by providing two distinct syntactic trees for the sentence (to avoid dealing with a lexicon, we can consider lexical categories (N, Det, Prep, V...) as terminal symbols).
2. Give an ambiguous CFG grammar capable of generating the two possible syntactic analyses.
3. Give a CFG in which the ambiguity is removed by assuming a systematic low attachment strategy, according to which prepositional phrases will get attached to the closest noun.

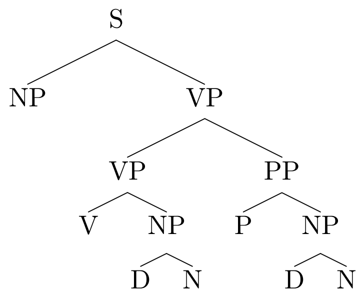
Answer

Let us assume the following "lexical" categories for this exercise:

(2) Sam saw a girl with his telescope
NP V D N P D N

1. On the left-hand side, an analysis where the PP is adjoined to the NP; on the right-hand side, a version where it is adjoined to the VP.



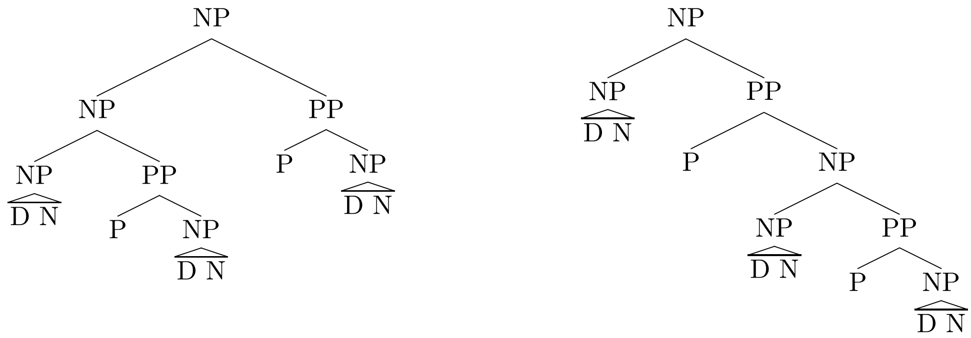


2. A typical CFG grammar covering the sentence would be the following one, where adjunction rules are used to introduce optional PPs
(an adjunction rule is a rule of the form XP $\rightarrow$ XP YP).

An other option, less X-bar-like, would be to have ternary rules of the form VP $\rightarrow$ V NP PP.

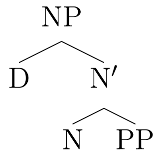
S	$\rightarrow$	NP	VP
NP	$\rightarrow$	PN	
	|	D	N
	|	NP	PP
PP	$\rightarrow$	P	NP
VP	$\rightarrow$	V	NP
	|	VP	PP

3. An easy option to just get rid of the right hand-side analysis above would be to remove the last rule in the previous grammar. However, this would not warrant that in the case of embedded NPs we would get only the low attachement strategy: a sequence like D N P D N P D N would give rise to two different structures:



The left-hand analysis would be appropriate for a phrase like *the mastery of the language by this teacher*, while the second structure would be more appropriate for a phrase like *the cat of the neighbour of my brother*.

To ensure that a low attachment strategy always prevail, we need to find a way to attach PPs very low. An option would be to have rules of this format:



(We need also to have a singleton rule N′ → N)

Predicate Logic

Ex. 5

Translate as precisely as possible the following sentences intro predicate logic. In case of ambiguity, provide a formula for each possible reading.

(3) a. An accomplished person doesn't talk unless everyone listens.
b. Not all the guests appreciated a singer.
c. Everytime Mia finds a wallet she gives it back to its owner.
d. At least one person has to come to make Sam happy.

. . . . Answer . . . .

(3a) *An accomplished person doesn't talk unless everyone listens.*

Let's focus on the propositional structure of the sentence. Let's represent a simpler sentence like (4a) as *not p unless q* (with p = *Sam talks* and q = *Jill is away*). (4a) is interpreted as a (one-way) conditional: *if Sam talks then necessarily Jill is away*; or, conversely *if Jill is not away, then Sam doesn't talk*; or *either Sam doesn't talk or Jill is away*. This corresponds to the logical structure in (4b). Consequentely, the sentence (4c) will get the representation in (4d). If you think that the intransitive use of *listens* should be made explicit (something like (4e)) then the representation (4f) should be adopted.

The indefinite at the beginning of the sentence is preferably interpreted as a generic or universal sentence: any person, if they are accomplished, are such that they don't talk unless everyone listens. The representation for this interpretation is given in (4g).

Even though I find it hardly available, an other reading could be that the indefinite is an existential (*there is an accomplished person who...*). In that case, a possible representation is (4h).

(4) a. Sam doesn't talk unless Jill is away. [*not p unless q*]
b. $(p \to q)$ or $(\neg q \to \neg p)$ or $(\neg p \lor q)$
c. x doesn't talk unless everyone listens
d. $(Tx \to \forall y\,(Py \to Ly))$
e. x doesn't talk unless everyone listens to x
f. $(Tx \to \forall y\,(Py \to Lyx))$
g. $\forall x\,(Px \to (Ax \to (Tx \to \forall y\,(Py \to Ly))))$
h. $\exists x\,(Px \land (Ax \land (Tx \to \forall y\,(Py \to Ly))))$

(3b) *Not all the guests appreciated a singer.*
This sentence contains 3 elements that may give rise to scope interaction effects: the negation, the universal quantification expressed by *all*, and the existential quantification expressed by *a*. The surface structure corresponds to negation having the largest scope, over universal quantification which itself has wide scope over existential quantification. This interpretation ("congruent", schematized $\neg\forall\exists$) can be paraphrased as in (5a), and represented as (5b), which is equivalent to (5d) and (5e) (naturally paraphrased as in (5c)).

If the existential quantification takes wide scope over both negation and universal quantification, we get a different interpretation ($\exists\neg\forall$) which still seems plausible. The paraphrase is in (5f), and possible formulae are given in (5g) and (5h).

In theory, one could consider a scope interaction where negation keeps a wide scope, but the existential and universal quantifications inverse their scope ($\neg\exists\forall$). A paraphrase for this reading would be (5i), with the formula in (5j). However I'm not sure that this reading is really available to speakers, any more than the 3 other possible orders ($\forall\neg\exists$, $\forall\exists\neg$, or $\exists\forall\neg$). One reason could be that *not* and *all* can't be separated in such a sentence.

(5) a. *It is not the case that all the guests appreciated at least one singer*
b. $\neg\forall x\,(Gx \to \exists y\,(Sy \land Axy))$
c. *Some guests did not appreciate any singer*
d. $\exists x\,(Gx \land \neg\exists y\,(Sy \land Axy)$
e. $\exists x\,(Gx \land \forall y\,(Sy \to \neg Axy)$
f. *A singer is such that not all the guests appreciated them*
g. $\exists y\,(Sy \land \neg\forall x\,(Gx \to Axy))$
h. $\exists y\,(Sy \land \exists x\,(Gx \land \neg Axy))$
i. *It is not the case that there is a singer that is appreciated by all the guests*
j. $\neg\exists y\,(Sy \land \forall x\,(Gx \to Axy))$

(3c) *Everytime Mia finds a wallet she gives it back to its owner*
Since we don't deal with temporal and aspectual aspects of meaning in our framework, I suggest that the initial sentence be reformulated as in (6a), which is equivalent to (6b), which makes it more visible that we deal with a donkey sentence. A compositional "translation" of (6b) leads to an open formula (6c), which can't be taken as representing the meaning of the initial sentence. The representation in (6d) is correct. While *its owner* could be paraphrased as *the owner of it*, which we would have trouble translating into logic, I translate the expression as *an owner of it*. Alternatively, one could chose to paraphrase it as *any owner of it*, in which case the representation is (6e). N.B. Here *Gabc* corresponds to *a gives b to c*.

(6) a. *Mia gives back every wallet she finds to its owner*
b. *If Mia find a wallet, she gives it back to its owner*
c. $(\exists x\,(Wx \wedge Fmx) \rightarrow \exists y\,(Oy\underline{x} \wedge Gm\underline{x}y))$
d. $\forall x\,((Wx \wedge Fmx) \rightarrow \exists y\,(Oyx \wedge Gmxy))$
e. $\forall x\,((Wx \wedge Fmx) \rightarrow \forall y\,(Oyx \rightarrow Gmxy))$

(3d) *At least one person has to come to make Sam happy*
The statement gives a necessary condition for Sam to be happy, namely that at least one person comes. The proper representation of A being a necessary condition for B uses a conditional $B \rightarrow A$: if B came about then its necessary condition is fullfilled. Therefore the correct representation for (3d) is (7a).

(7) a. $(Hs \rightarrow \exists x\,(Px \wedge Cx))$

Montague Programme (« Grammar engineering »)

Ex. 6

(A) Propose a grammar and semantic rules (or a decorated syntax tree) for a fragment fully defined syntactically and compositionally that contains (8a) and also (8b).

(8) a. Bob sees a farmer.
    b. A farmer sees Bob.

(B) Extend the previous fragment in the most straightforward way to include noun phrases with subject relative clauses as illustrated in (9a). Make sure the fragment is general enough to also include (9b).

(9) a. Bob sees a farmer who dances.
    b. Bob sees a farmer who sees Bob.

(C) To account for object relative clauses like the one in (10), we will assume that while *Pam sees Bob* denotes a full proposition Spb, the relative clause *Pam sees _* denotes a predicate, namely $\lambda x.\ Spx$. Extend the fragment to include object relative clauses.

(10) Bob sees a farmer whom Pam sees.

(D) Extend the fragment to include (11) (which is not a donkey sentence).

(11) Every farmer who owns a donkey dances.

(E) Extend the fragment with conditional sentences of the form *if P then Q*, and verify that (12) can be accounted for in the fragment.

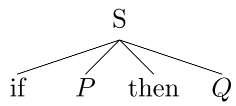
(12) If a farmer owns a donkey then Bob dances.

(F) What would be an appropriate logical formula for (13)? (assuming *he* is anaphoric to *a farmer*.)

(13) If a farmer owns a donkey then he dances.

Assuming that *he* denotes (magically) the right free variable, verify that it is possible to produce compositionaly the right representation.

(G) Assuming the following syntax tree for a conditional sentence, and that a conditional sentence is "translated" into a material implication, what are the types of the denotations of P and Q?



Suppose now that we focus on conditional sentences with an indefinite NP inside the antecedent like (14), so that we can use the equivalence between $(\exists x\ \varphi \to \bar{\beta})$ and $\forall x\ (\varphi \to \bar{\beta})$ (with $\bar{\beta}$ not containing free occurrences of x). What would be the types of the denotations of P and Q if we were to produce compositionally the universal variant?

(14) If Bob sees a farmer then Bob dances.

(H) [Bonus] Provide a fragment that can produce compositionaly the appropriate representation for the real donkey sentence (15). It is of course not expected that the fragment will not break on other similar sentences.

(15) If a farmer owns a donkey then he beats it.

. . . . Answer . . . .