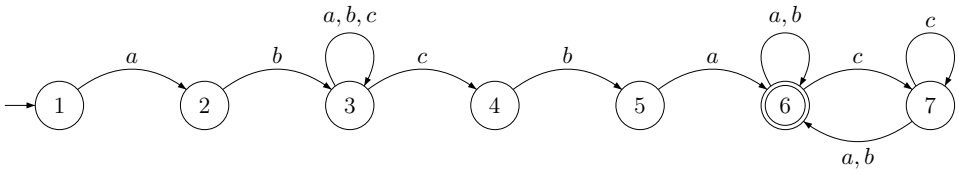
Ex. 1

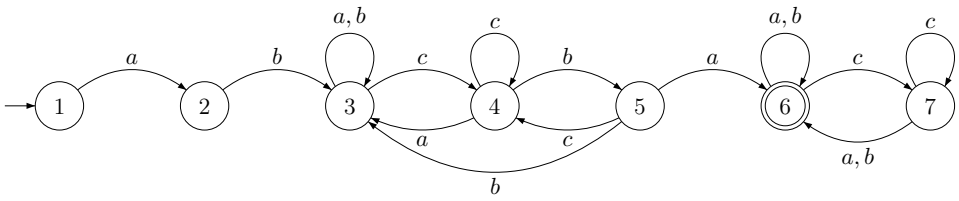
Propose a deterministic finite state automaton which recognizes all the words on Σ^* which start with the prefix *ab*, include the factor *cba*, and do not end with *c*.
$\Sigma = \{a, b, c\}$

Answer

We may want to start with a non-deterministic version. The states from 1 to 3 deal with the prefix *ab*, the states 3 to 6 deal with the factor *cba* (which may not immediately follow the prefix), the last condition (not ending with *c*, which is equivalent to ending with either *a* or *b*) is dealt with by states 6 and 7. Notice that since the factor *cba* doesn't end with *c*, a word ending with this factor should be accepted.



To get a deterministic version we have to deal with the only non-unary transition of the previous automaton ($\delta(3, c) = 3$ or 4).



Note: To make this automaton complete an additional state (« well ») is necessary, as well as transitions to this state from states 1 and 2.

Ex. 2

Propose a complete deterministic finite state automaton which recognizes all the words on Σ^* such that all c's are before all b's (if any), the number of c's is odd (thus $\geqslant 1$) and the number of a's is even, and b's can occur only if they are not followed by a's ($\Sigma = \{a, b, c\}$).

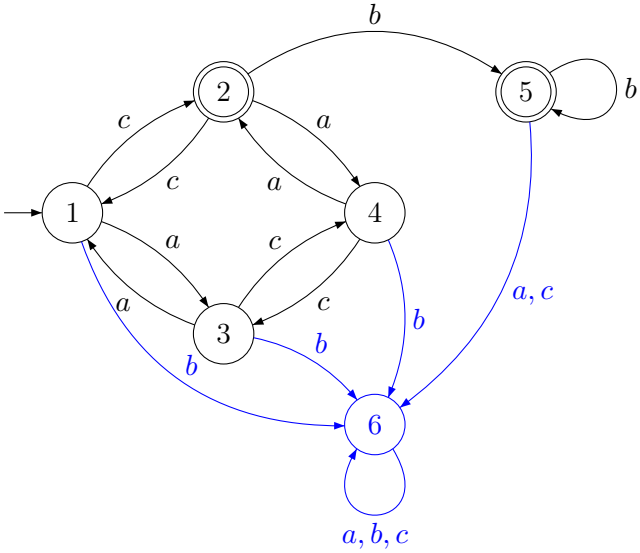
Answer

The conditions on accepted words are reformulated here:

1. all c's before b's
2. b's not followed by a's
3. odd number of c's ($\geqslant 1$)
4. even number of a's ($\geqslant 0$)

Conditions 1 and 2 together entail that only b's can follow b's. In other words, as soon as a b is read, only additional b's can be read.
We can now focus on the two remaining conditions. There are exactly four different configurations depending on the evenness of the numbers of a's and c's, we associate a state to each of them, transitions can be defined accordingly. The only favorable situation corresponds to state 2. The state number 6 is a "well" state, corresponding to all the cases where a b occurs at the wrong place or either a or c occurs after a b was read.

a	c	state
even	even	1
even	odd	2
odd	even	3
odd	odd	4



Ex. 3

Propose a deterministic finite state automaton which recognizes the language L, the set of all the words of length $\leqslant 4$ which are formed by the concatenation of two identical factors ($\Sigma = \{a, b, c\}$).
$L = \{w \in \Sigma^* \mid \exists u \in \Sigma^*,\ w = uu \ \&\ |w| \leqslant 4\}$.

Answer

"Copy words" necessarily have an even number of letters, so we expect the language to contain ε, 3 two-letter words (aa, bb, cc). All four-letter words will be formed out of two copies of one two-letter word. Since there are 9 (3^2) different two-letter words in Σ^*, we end up with 9 four-letter words in L.
The most legible version is on the left, while a minimal version is on the right. Making those automata complete would lead to a large number of additional transitions going to a « well » state.