$S \to p \mid q \mid r \mid s \ldots$

$S \to (S \text{ OP } S)$

$\text{OP} \to \wedge \mid \vee \mid \to$

$S \to \neg S$

$\wedge$ and

$\vee$ or

$\to$ arrow

$\neg$ not

$$(p \wedge (q \to r))$$

Propositional logic

$$(((p \wedge q) \to r) \vee \neg p)$$

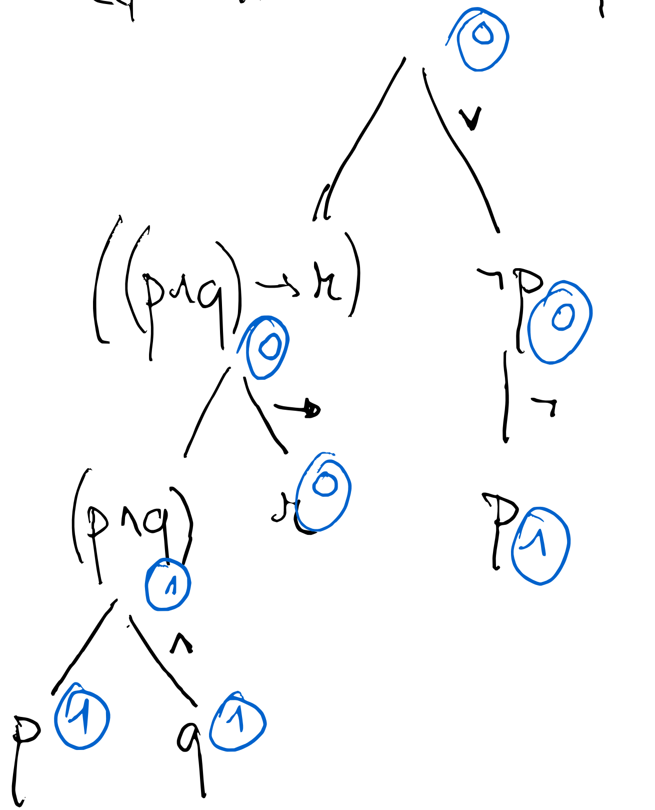
$$((x + y) - z)$$

$$((3+7) - 2)$$

x	y		z
1	1	1	1
2	1	1	2
3	7	242	...
⋮	⋮		

p	q
0	0
0	1
1	0
1	1

$((p \wedge q) \to r) \vee \neg p)$



$p = 1$
$q = 1$
$r = 0$

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

p	$\neg p$
0	1
1	0

p	q	$(p \vee q)$
0	0	0
0	1	1
1	0	1
1	1	1

$p \wedge q \to r$

$((p \wedge q) \to r)$

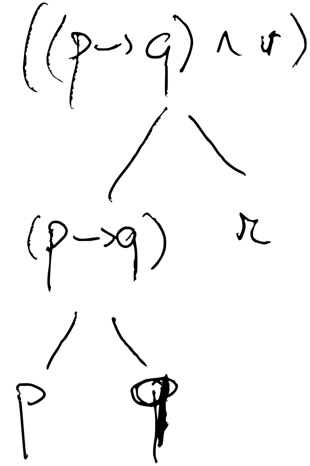
$(p \wedge (q \to r))$

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

if it rains,

the road is wet.

$((p \to q) \wedge r)$



2^k

p	q	r	$(p \to q)$	$((p \to q) \wedge r)$
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	0	0
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

k variables.

$(p \wedge \neg p)$

contradiction

tautology.

contingent formula.

p	$\neg p$	$(p \wedge \neg p)$	$(p \vee \neg p)$
0	1	0	1
1	0	0	1

$(p \rightarrow q)$ $(x + y)$

Relations between formulae

φ & χ are logically equivalent

$\varphi = (p \to q)$

$\psi = \neg(\neg p \wedge q)$ $\qquad$ $\chi = \neg(p \wedge \neg q)$

		φ			ψ			χ
p	q	$(p \to q)$	$\neg p$	$(\neg p \wedge q)$	$\neg(\neg p \wedge q)$	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$
0	0	1	1	0	1	1	0	1
0	1	1	1	1	0	0	0	1
1	0	0	0	0	1	1	1	0
1	1	1	0	0	1	0	0	1

$(p \to q)$

ψ is a logical conseq. of φ

iff

everytime φ is true, ψ is also true.

$$\frac{\begin{array}{c}(p \to q)\\ p\end{array}}{q}$$

modus ponens

$\varphi : ((p \to q) \wedge p) \qquad \psi : q$

p	q	$(p \to q)$	φ: $((p \to q) \wedge p)$	ψ: q	$(\varphi \to \psi)$: $(((p \to q) \wedge p) \to q)$
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	0	1
1	1	1	1	1	1

Deduction theorem

$\varphi \models \psi$ ψ is a logical cq of φ

iff.

$\models (\varphi \to \psi)$ the formula $(\varphi \to \psi)$ is a tautology

$(\varphi \to \psi)$ is a tautology

p	q	$(p \leftrightarrow q)$
0	0	1
1	1	1
0	1	0
1	0	0

φ and ψ are logically equivalent

iff

$(\varphi \leftrightarrow \psi)$ is a tautology

The engine is not noisy

but it uses lots of gas.

p : the engine is noisy

q : the engine uses lots of gas

$(\neg p \wedge q)$ $\qquad$ $(\neg \cancel{p} \rightarrow q)$

p	q
0	0
0	1
1	0
1	1